**Terminal Velocity Simulation Conducted on a Spreadsheet**

**Objective:** To write a program that will calculate terminal velocity for various objects on a spreadsheet given their cross-sectional area, mass, and drag coefficient.

**Discussion:** Students are often taught to disregard air resistance in any problem involving freefall. For massive objects at low speeds, this assumption is good. However, in reality air resistance begins affecting the motion of objects at relatively low speeds and must be considered to accurately model the motion of an object in freefall. When an object encounters air resistance as it falls, its acceleration begins to drop below 9.8 m/sec2.

As a falling object’s velocity increases the drag force it experiences increases and approaches the weight of the object. When these two forces become balanced (drag force = weight) and the object stops accelerating. When this happens, the object has reached its **terminal velocity**.

Since drag force (Fd) is proportional to the cross-sectional area (CSA) of an object and proportional to the velocity squared (v2), both must be considered in calculating it. To solve this analytically would require calculus (differential equations) which can be avoided using a spreadsheet that breaks up the motion into many small intervals which can be assumed to have uniform acceleration making the following equations valid.

**Governing Equations:**

** Eq. 1.1**

** Eq. 1.2**

** Eq. 1.3**

 ** Eq. 1.4**

where **FD** is drag force; **c** is the drag coefficient; **A** is object cross-sectional area; **v** is velocity; **w** is object weight; **m** is object mass; **g** is 9.8 m/sec2; **a** is acceleration; **Fnet** is net force; **t** is time increment.

**Setting up the spreadsheet:** Open Sheets on Google Docs.

1. Set up your spreadsheet **exactly** as it appears below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | A | B | C | D | E | F |
| 1 | Mass (kg) | CSA (m2) | Drag Coefficient | Time Increment |  |  |
| 2 |   |   |   |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 | Time (sec) | Velocity (m/sec) | Weight (N) | Drag Force (N) | Net Force (N) | Acceleration (m/sec^2) |

1. The time increment will be given. It varies with the object being viewed. Objects that reach their terminal velocity quickly like a piece of paper or feather will require shorter time increments. Start with a time increment of 0.5 seconds which can be entered into cell D2 (do not include units). In cell A5, type “0”. In cell A6 type “ =$D$2+A5” **(Do not include the quotation marks**) Select cell A6 and then grab the fill handle in the lower right hand corner of the cell and drag down to cell 95. Column A should now count time by increments specified in D2.
2. In cell B5 type “0” for the initial velocity. In cell B6 type “=B5+F5\*$D$2” to calculate the new velocity from Eq. 1.4.
3. Using Eq. 1.2 in cell C5 type “=9.8\*$A$2” for the weight of the object.
4. In cell D5 type “=$C$2\*$B$2\*B5^2”. This is Eq. 1.1 the equation for drag force (area times velocity squared times drag coefficient.)
5. In cell E5 type “=C5-D5” to sum the forces (weight and drag force).
6. In cell F5 type “= E5/$A$2” to calculate acceleration of the object using Newton’s 2nd Law (Eq. 1.3). Entering a mass will prevent a “divide by zero” error. Enter “95” in cell A2 for the mass of the first object.
7. Now your spreadsheet is almost complete. Go to the fill handle in the lower right hand corner of the bottom cell containing a formula in each column and double click each fill handle (starting from Column B) so the columns fill in to cell 95.
8. Enter the data (CSA and Drag Coefficient) in for the skydiver found in table 1.1 and verify that the drag force is now calculating and the acceleration is decreasing.
9. Create a graph of the velocity (Y) vs. time (X) for the object in question.
	1. To create a graph click on any empty/blank cell and then open the menu from the insert tab at the top of the page.
	2. Choose "Chart" from the insert menu 
	3. Make sure to be on the Setup tab at the top of the chart editor (see figure on next page). Under the "Chart type" dropdown menu, use the scroll bar to move down and select "scatter" which can be found in the window (labeled c.) along with choices of all other possible chart types listed. **Note:** do not use a line graph for mathematical data/graphs.
	4. To select the data to graph, first choose the Y values to be graphed and click on the grid icon in the series tab (labeled d with the arrow pointing to it in the figure) and then highlight all of the column B data for the velocity including the column data label and then click OK or press enter.

c.

d.

e.

* 1. Next you will choose the X values vs which the Y values will be graphed. To do so click on the grid icon in the X-axis tab (labeled e with the arrow pointing to it in the chart editor figure) and highlight all of the column A data for the time (including the column data labels) and then press enter.
	2. Your graph should be complete. To add a data series to an existing graph you can only do so if you have data that has all identical X values in Google Sheets. This is a limitation in some cases but not in this case as only one data set is being graphed as an XY scatterplot.

Confirm the formulas match below and have the same number of data points in X and Y fields.

X values field (e.): 'Terminal Velocity'!A4:A95

Y values field (d.): 'Terminal Velocity'!B4:B95

**Analysis:**

Step 9 instructed you to enter the data for the skydiver’s cross-sectional area and drag coefficient in cells B2 and C2 to find the terminal velocity of the skydiver by scrolling down in column B (the remaining items are listed in table 1.1 below).

1. Predicting terminal velocity: **before inputting the values** given in the table on the next page, try to guess the terminal velocity of each of the items. You will not be penalized for incorrect predictions. Estimate as best you can and write your guesses into the table in the “Predicted Terminal Velocity” column. Then use the spreadsheet to determine the actual according to the model (step 2)
2. **Only one object can be viewed at a time**. Now enter the data for a single object from the table of objects, their masses, cross-sectional areas, drag coefficients and time increments which are to be entered into cells A2 thru D2 on the spreadsheet to determine the terminal velocity. Scroll down to see the velocity level off and when it is holding constant it is an acceptable result. Or you may simply go to the bottom cell (B95) and look at that value. Observe the progression of the net force and acceleration columns as the object is speeding up over time.
3. Record the actual terminal velocity for each item in the table below.

**Table 1.1**: Objects to be simulated falling through air to reach terminal velocity. **Guess first and record in the predicted column. Then input the object specs and observe the actual terminal velocities and record each one in the table below.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Falling Item | Mass (kg) | CSA (m^2) | Drag Coefficient | Time Increment (sec) | Predicted Terminal Velocity (m/sec) | Actual Terminal Velocity (m/sec) |
| Skydiver | 95 | 0.6 | 0.55 | 0.5 |  |   |
| skydiver (head first dive) | 95 | 0.12 | 0.55 | 1 |  |   |
| skydiver (open parachute) | 95 | 25 | 0.5 | 0.1 |  |   |
| Feather | 0.001 | 0.002 | 0.55 | 0.02 |  |   |
| Javelin | 0.8 | 0.0004 | 0.45 | 1.5 |  |   |
| penny (sideways) | 0.003 | 0.0000019 | 0.5 | 1 |  |   |
| penny (facing down) | 0.003 | 0.0002834 | 0.55 | 0.1 |  |   |
| piece of paper | 0.005 | 0.0616 | 0.6 | 0.01 |  |   |
| Bullet | 0.013 | 0.0000785 | 0.45 | 0.5 |  |   |

**Development Questions:**

1. This spreadsheet assumes uniform acceleration over the specified time interval. For this to hold true, what must be true about the size of the time interval?
2. Why is it useful to be able to change the time increment at the top of the spreadsheet?
3. What do the dollar signs in each formula mean?
4. How is the net force on the object determined and how does it relate to acceleration?

**Analysis Questions:**

1. Why does a skydiver travel different speeds in the different conditions?
2. Describe the effect of increasing or decreasing each of the following on an objects terminal velocity holding other factors constant. a) mass b) cross-sectional area (CSA) c) drag coefficient
3. The Returning Bullet: If a bullet was fired straight up, it would slow down to a velocity of zero and fall back toward earth. Would it be as deadly on its way down if it left the gun at 350 meters per second?
4. The actual terminal velocity of a rotating penny varies. Why?
5. At the beginning of each graph, there is a region that appears to be linear. What is the approximate slope of that region? Explain your answer.
6. If an object was traveling straight up in air, how would its acceleration compare to 9.8 m/sec2? If it was falling straight down, how would its acceleration compare to 9.8 m/sec2? Assume gravity and air resistance are the only forces acting on it.
7. Choose 3 sports implements (like baseballs, golf balls etc.) and look up or measure their standard masses and diameter (D) to calculate their cross-sectional areas, assume a drag coefficient of 0.45 and determine their terminal velocities.



1. Determine the diameter of a rain drop in cm if it has a mass of 2.9\*10-8 kg if its terminal velocity is 1.03 meters per second. Use a drag coefficient of 0.6 and a time increment of .005. *Hint: start with a small CSA, using trial and error find the CSA that produces the correct terminal velocity, then use solve for diameter (D) with the equation provided above.*