**Terminal Velocity Simulation Conducted on a Spreadsheet**

**Objective:** To write a program that will calculate terminal velocity for various objects on a spreadsheet given their cross-sectional area, mass, and drag coefficient.

**Discussion:** Students are often taught to disregard air resistance in any problem involving freefall. For massive objects at low speeds, this assumption is good. However, in reality air resistance begins affecting the motion of objects at relatively low speeds and must be considered to accurately model the motion of an object in freefall. When an object encounters air resistance as it falls, its acceleration begins to drop below 9.8 m/sec2.

As a falling object’s velocity increases the drag force it experiences increases and approaches the weight of the object. When these two forces become balanced (drag force = weight) and the object stops accelerating. When this happens, the object has reached its **terminal velocity**.

Since drag force (Fd) is proportional to the cross-sectional area (CSA) of an object and proportional to the velocity squared (v2), both must be considered in calculating it. To solve this analytically would require calculus (differential equations) which can be avoided using a spreadsheet that breaks up the motion into many small intervals which can be assumed to have uniform acceleration making the following equations valid.

**Governing Equations:**

** Eq. 1.1**

** Eq. 1.2**

** Eq. 1.3**

 ** Eq. 1.4**

where **FD** is drag force (N); **c** is the drag coefficient (kg/m3); **A** is object cross sectional area (m2); **v** is velocity (m/s); **w** is object weight (N); **m** is object mass (kg); **g** is 9.8 m/sec2; **a** is acceleration (m/s2); **Fnet** is net force (N); **t** is time increment (sec).

**Setting up the spreadsheet:** Open Microsoft EXCEL.

1. Set up your spreadsheet **exactly** as it appears below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | A | B | C | D | E | F |
| 1 | Mass (kg) | CSA (m2) | Drag Coefficient (kg/m3) | Time Increment (sec) |  |  |
| 2 |   |   |   |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 | Time (sec) | Velocity (m/sec) | Weight (N) | Drag Force (N) | Net Force (N) | Acceleration (m/sec^2) |

1. The time increment will be given. It varies with the object being viewed. Objects that reach their terminal velocity quickly like a piece of paper or feather will require shorter time increments. Start with a time increment of 0.5 seconds which can be entered into cell D2 (do not include units). In cell A5, type “0”. In cell A6 type “ =$D$2+A5” **(Do not include the quotation marks**) Select cell A6 and then grab the fill handle in the lower right hand corner of the cell and drag down to cell 95. Column A should now count time by increments specified in D2.
2. In cell B5 type “0” for the initial velocity. **(Do not include the quotation marks.)** In cell B6 type “=B5+F5\*$D$2” to calculate the new velocity from Eq. 1.4.
3. Using Eq. 1.2 in cell C5 type “=9.8\*$A$2” for the weight of the object.
4. In cell D5 type “=$C$2\*$B$2\*B5^2”. This is Eq. 1.1 the equation for drag force (area times velocity squared times drag coefficient.)
5. In cell E5 type “=C5-D5” to sum the forces (weight and drag force).
6. In cell F5 type “= E5/$A$2” to calculate acceleration of the object using Newton’s 2nd Law (Eq. 1.3). Entering a mass will prevent a “divide by zero” error. Enter “95” in cell A2 for the mass of the first object.
7. Now your spreadsheet is almost complete. Go to the fill handle in the lower right-hand corner of the bottom cell in each column and double click each fill handle (starting from the left) so the columns fill in down to cell 95.  ***Be sure to click the lower formula containing cell in any given column.***
8. Create a graph of the velocity (Y) vs. time (X) for the object in question.

 a. Click on the Insert tab at the top of the screen.

b. Select the Scatter icon (looks like dots on a coordinate axis) and choose the graph type without the data points on it

c. To select the data **right click** the chart area and choose “Select Data” from the drop down menu. Remove any data series that is automatically selected. Next click the icon (below the arrow) to the right of the formula field for the X values. Once you do this the spreadsheet will be active and you can highlight cells A5 to A95. **Do not highlight column headings, just data points.** Then click the same icon next to the formula field to return to the data series window.

e. Do the same for the Y values (B5 to B95).

f. If you wish to move the graph after the data has been selected, right click the chart area and select “Move Chart” which will allow you to put it into a new sheet. This is recommended because it will allow you to place it out of the way of your active sheet and has a convenient chart tab at the bottom next to your sheet tabs.

g. Adding Axes: Select the layout tab and choose an option that allows the horizontal and vertical axes to be created and name them time (seconds) and velocity (m/s) respectively.

\*If Excel 2003 is used refer to the instructions at the end of the document.

**Analysis:**

Enter the data for the skydiver’s cross-sectional area and drag coefficient in cells B2 and C2 to find the terminal velocity of the skydiver. (items are listed in table 1.1 below)

1. Predicting terminal velocity: **before inputting the values** given in the table on the next page, try to guess the terminal velocity of each of the items. You will not be penalized for incorrect predictions. Estimate as best you can and write your guesses into the table in the “Predicted Terminal Velocity” column. Then use the spreadsheet to predict (step 2)
2. **Only one object can be viewed at a time**. Now enter the data for a single object from the table of objects, their masses, cross sectional areas, drag coefficients and time increments which are to be entered into cells A2 thru D2 on the spreadsheet to determine the terminal velocity. Scroll down to see the velocity level off and when it is holding constant it is an acceptable result. Or you may simply go to the bottom cell (B95) and look at that value. Observe the progression of the net force and acceleration columns as the object is speeding up over time.
3. Record the actual terminal velocity for each item in the table below.

**Table 1.1**: Objects to be simulated falling through air to reach terminal velocity.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Falling Item | Mass (kg) | CSA (m^2) | Drag Coefficient | Time Increment (sec) | Predicted Terminal Velocity (m/sec) | Actual Terminal Velocity (m/sec) |
| Skydiver | 95 | 0.6 | 0.55 | 0.5 |  |   |
| skydiver (head first dive) | 95 | 0.12 | 0.55 | 1 |  |   |
| skydiver (open parachute) | 95 | 25 | 0.5 | 0.1 |  |   |
| Feather | 0.001 | 0.002 | 0.55 | 0.02 |  |   |
| Javelin | 0.8 | 0.0004 | 0.45 | 1.5 |  |   |
| penny (sideways) | 0.003 | 0.0000019 | 0.5 | 1 |  |   |
| penny (facing down) | 0.003 | 0.0002834 | 0.55 | 0.1 |  |   |
| piece of paper | 0.005 | 0.0616 | 0.6 | 0.01 |  |   |
| Bullet | 0.013 | 0.0000785 | 0.45 | 0.5 |  |   |

**Development Questions:**

1. This spreadsheet assumes uniform acceleration over the specified time interval. For this to hold true, what must be true about the size of the time interval?
2. Why is it useful to be able to change the time increment at the top of the spreadsheet?
3. What do the dollar signs in each formula mean?
4. How is the net force on the object determined and how does it relate to acceleration?

**Analysis Questions:**

1. Why does a skydiver travel different speeds in the different conditions?
2. Describe the effect of increasing or decreasing each of the following on an objects terminal velocity holding other factors constant. a) mass b) cross sectional area (CSA) c) drag coefficient
3. The Returning Bullet: If a bullet was fired straight up, it would slow down to a velocity of zero and fall back toward earth. Would it be as deadly on its way down if it left the gun at 350 meters per second?
4. The actual terminal velocity of a rotating penny varies. Why?
5. At the beginning of each graph, there is a region that appears to be linear. What is the approximate slope of that region? Explain your answer.
6. If an object was traveling straight up in air, how would its acceleration compare to 9.8 m/sec2? If it was falling straight down, how would its acceleration compare to 9.8 m/sec2? Assume gravity and air resistance are the only forces acting on it.
7. Choose 3 sports implements (like baseballs, golf balls etc.) and look up or measure their standard masses and diameters to calculate their cross-sectional areas, assume a drag coefficient of 0.45 and determine their terminal velocities.
8. Determine the diameter of a rain drop in cm if it has a mass of 2.9\*10-8 kg if its terminal velocity is 1.03 meters per second. Use a drag coefficient of 0.6 and a time increment of .005. *Hint: start with a small CSA, use goal seek, then solve for diameter.*

**Making a Graph on Excel 2003 or earlier:**

* 1. To create a graph, click on the chart wizard (looks like a little bar graph in the tool bar up top).
	2. Choose X-Y scatter for your chart type and click next.
	3. Click on the series tab up top, remove any series that are there and then select add series.
	4. Click the icon next to the field and highlight cells A5 to A95. You could also click on field next to the X values and type “ =Sheet1!$A$5:$A$95”.
	5. Click on field next to the Y values and type “ =Sheet1!$B$5:$B$95”
	6. Label your chart based on the object you use and axes velocity and time, and then select the location you would like to place your chart and click OK.